Magnetic phase transition and confinement regimes

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Inspiration and motivation

• Since the discovery of the H-mode, many models of the L-H transition have been proposed

• The now conventional model is based on electrostatic turbulent vortices shredded by rotation shear. It is highly developed, can include sophisticated predator-prey model, it is now beginning to move towards electromagnetic consequences of electrostatic fluctuations…

• It is a very attractive model to many, but then I saw this movie of an L-H transition in MAST

  http://www.ccfe.ac.uk/videos.aspx?currVideo=24&currCateg=0

  (L-mode 10-18 s, H-mode later)

and I started thinking about phase transitions.

  One of the better studied phase transitions in physics is the magnetic phase transition.

  So that got me thinking some more…
Usual set-ups

Microturbulence

\[ \gamma > 0 \]

electrostatic
drift velocity
Sophisticated, non-linear models
\[ \lambda \sim (1-10) \rho_s \]

MHD

\[ \gamma > 0 \]
electro-magnetic
\[ V_{\text{thermal}} \text{ or } V_{\text{Alfvén}} \]
Linear models
Energy principle
\[ \lambda \sim \frac{p}{p'} \text{ or } \lambda \sim a \]
Microturbulence

\[ \gamma > 0 \]

electrostatic

drift velocity

Sophisticated, non-linear models

\[ \lambda \sim (1-10) \rho_s \]

MHD

\[ \gamma > 0 \]

electromagnetic

“magnetic test-particle”

subcritical

meso-scale

\[ V_{\text{thermal}} \text{ or } V_{\text{Alfvén}} \]

Linear models

Energy principle

\[ \lambda \sim p/p' \text{ or } \lambda \sim a \]
Index

- Plasma overall magnetisation (cylindrical tokamak approximation)
- Magnetisation of “tubes” (field aligned pressure perturbations)
- Motion of magnetised tubes in magnetised plasma ($B_z$ gradient)
- Effect on profiles
- Connections to interchange stability theory
- Experimentally testable criterion
- Data!

References:
Plasma Equilibrium

Plasma force balance:

\[ \nabla p = j \times B = j_\zeta \times B_\theta + j_\theta \times B_\zeta \]

\[ p' \equiv \frac{dp}{d\Psi} \]

\[ j_\zeta = -\left( R p' + FF'/(\mu_0 R) \right) \]

\[ \mu_0 j_\theta = -F'B_\theta , \quad F(\Psi) = RB_\zeta \]

In cylindrical approximation:

\[ \frac{d}{dr} \left( p + \frac{B_z^2 + B_\theta^2}{2\mu_0} \right) = -\frac{B_\theta^2}{r\mu_0} \]

\[ j_z = -\left( R_0 p' + FF'/(\mu_0 R_0) \right) \]

\[ \mu_0 j_\theta = -\frac{dB_z}{dr} \]
Plasma magnetization of a “cylindrical” tokamak

Integrating cylindrical force balance:

\[ \beta_\theta = \frac{\int_0^a pdS}{B_{\theta a}^2 / 2\mu_0} = \frac{B_{za}^2 - \langle B_z^2 \rangle}{B_{\theta a}^2} \approx 1 + \frac{2B_{za} \left( B_{za} - \langle B_z \rangle \right)}{B_{\theta a}^2} \]

\[ \frac{dB_z}{dr} = -\mu_0 j_\theta \]

\( (\beta_\theta - 1) \) is related to normalised average plasma magnetisation

- \( \beta_\theta < 1 \)  
  B\(_z\) increased by \( j_\theta \)  
  paramagnetism, low pressure

- \( \nabla p < \vec{j}_\zeta \times \vec{B}_\theta \)

- \( \beta_\theta > 1 \)  
  B\(_z\) reduced by \( j_\theta \)  
  diamagnetism, high pressure

- \( \nabla p > \vec{j}_\zeta \times \vec{B}_\theta \)
The tokamak plasma is a magnet.

\[ \langle B_z \rangle - B_{z\text{vac}} \approx \mu_0 \left( \frac{B_{\theta a}^2}{2\mu_0} - \int_0^a pdS \right) / B_{z\text{vac}} \]

the difference between poloidal magnetic and kinetic pressure determines if it is a para-magnet or a dia-magnet

**Paramagnets**
- increase the background magnetic field
- move towards high field regions

**Diamagnets**
- decrease the background magnetic field
- move towards low field regions

so far I have just reviewed well known facts
Diamagnetic levitation

A frog (diamagnetic) dropped in a strong magnetic field levitates because it tries to get away from the high field. It moves towards the lower field, arranged to be upwards.

M V Berry and A K Geimz
Magnetised plasma element

The poloidal current density around a tokamak plasma is responsible for plasma magnetisation: the difference between the externally applied toroidal field and the local toroidal field inside the plasma.

Not conventional current “filaments”, with parallel current density

Next consider a field-aligned plasma element with a pressure perturbation relative to the background plasma pressure.

What is its magnetisation?
Magnetism in cylindrical tube with pressure hill/hole

\[ F_\rho = mn \frac{dv_\rho}{dt} = -\nabla_\rho \tilde{p} + (\tilde{j} \times B)_\rho \]

\[ \tilde{j}_{\perp} = \frac{b \times \nabla \tilde{p}}{B} \]

Diamagnetic current:
if inside the tube there is a pressure hill (more pressure than in the background plasma), the associated perpendicular current reduces \( B_z \): diamagnetism
Magnetism in cylindrical tube with pressure hill/hole

\[ F_\rho = mn \frac{dv_\rho}{dt} = -\nabla_\rho \tilde{p} + (\tilde{j} \times \tilde{B})_\rho \quad \tilde{j}_\perp = \frac{b \times \nabla \tilde{p}}{B} \]

**Diamagnetic current:**
- if inside the tube there is a pressure hill, the associated perpendicular current reduces \( B_z \): diamagnetism

**Paramagnetic current:**
- if inside the tube there is a pressure hole (less pressure than in the background plasma), the associated perpendicular current increases \( B_z \): paramagnetism
Magnetism in cylindrical tube with pressure hill/hole

\[ F_\rho = m n \frac{d\mathbf{v}_\rho}{dt} = -\nabla_\rho \tilde{p} + \left( \tilde{j} \times \mathbf{B} \right)_\rho \quad \tilde{j}_\perp = \frac{\mathbf{b} \times \nabla \tilde{p}}{B} \]

Diamagnetic current:
if inside the tube there is a pressure hill, the associated perpendicular current reduces \( B_z \): diamagnetism

Paramagnetic current:
if inside the tube there is a pressure hole, the associated perpendicular current increases \( B_z \): paramagnetism

Magnetization of the blob:

\[ \nabla \times \tilde{\mathbf{M}} = \mu_0 \frac{\mathbf{b} \times \nabla \tilde{p}}{B} = -\frac{d\tilde{\mathbf{M}}}{dr} \hat{\mathbf{r}} \]

\[ \tilde{\mathbf{M}} = \frac{1}{\lambda_\parallel} \int_0^\rho \frac{\mathbf{b}}{B} \frac{\partial \tilde{p}(\rho')}{\partial \rho'} \lambda_\parallel d\rho' \approx -\frac{\tilde{p}}{B} \mathbf{b} \]

\( \{ \begin{array}{l} < 0, \text{ dia} \\ > 0, \text{ para} \end{array} \)
Movement of magnetized object in field gradient
(see Jackson)

\[ m_v \frac{dv}{dt} \bigg|_v = \int (\nabla (\tilde{M} \cdot B)) dV \approx \begin{align*}
\vec{B} &= \vec{B}_0 + \vec{r} \cdot \nabla \vec{B}_0 + ...
\end{align*} \]

\[ \approx (\int (\vec{r} \times \vec{j}) dV) \int \nabla B_{0z} dV \]

\[ mn_v \frac{dv}{dt} \approx \tilde{M}_z \nabla \tilde{B}_{z0} = -\mu_0 \tilde{M}_z \tilde{j}_\theta \]

the cold tube (paramagnetic) seeks high field

the hot tube (diamagnetic) seeks low field

averaged dB_z/dr controls motion of magnetised plasma tubes:

**Anti-potential** leads to **magnetic phase separation**
Paramagnetic plasma: L-mode

Motion of pressure blobs depends on $\frac{dB_z}{dr}$

$$mn_V \frac{dv_r}{dt} \approx \tilde{M}_\zeta \frac{d\tilde{B}_\zeta}{dr}$$

paramagnetic cold blobs move inward,
diamagnetic hot blobs move outward

outward thermal energy convection
at the expense of
inward magnetic energy convection

$\tilde{p}$ blobs “grow”, “instability”

L-mode
Diamagnetic plasma: H-mode

Motion of pressure blobs depends on $dB_z/dr$

$$mn V \frac{dv_r}{dt} \approx \bar{\bar{M}}_{\zeta} \frac{d\bar{B}_{\zeta}}{dr}$$

diamagnetic hot blobs move inward,
paramagnetic cold blobs move outward

inward thermal energy convection
at the expense of
outward magnetic energy convection

$\tilde{p}$ blobs “decrease”, “saturation”

H-mode
Magnetic Boundary: phase transition

$p(r)$ increases somewhere, creating diamagnetic region at plasma edge.
At a magnetic phase boundary blobs of the same type accumulate.

- **diamagnetic** blobs (heat) seek wells
- **paramagnetic** blobs seek hilltops

With multiple blobs moving, $p$ and $B_z$ profiles evolve.
Magnetic Boundary: phase transition

Pressure gradient
increases in diamagnetic region
Decreases in paramagnetic region

Magnetization,
of both signs, increases.

Phase transition is self-reinforcing.

Pressure pedestal forms, grows.

\[ p(r) \]

\[ B_z(r) \]
Evolution equations

Ideal 1 fluid MHD evolution

\[
\bar{n}_v m_i \frac{dv}{dt} = F \quad \text{magnetization force terms}
\]

\[
\frac{3}{2} \frac{\partial p}{\partial t} = - \nabla \cdot \mathbf{Q} + \mathbf{H}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times \eta \left( \mathbf{j} - j_{ni} \right)
\]

For now, consider what the magnetisation force does, disregard other transport mechanisms
Pedestal formation at magnetisation boundary

Assume dashed $B_z(r)$, $p(r)$ initial profiles

Ideal MHD with magnetization force

\[
\bar{n}_r m_i \frac{dv_r}{dt} \bigg|_M = \bar{M}_\zeta \frac{dB_z}{dr}
\]

\[
\frac{3}{2} \frac{\partial p}{\partial t} \bigg|_M = -\frac{d}{dr} (\tilde{p} v_r)
\]

\[
\frac{\partial B_z}{\partial t} \bigg|_M = \frac{d}{dr} (\tilde{v}_r B_{0z})
\]

Integrating one temporal step $\Delta t$

pressure steepens in diamagnetic regions,
increases diamagnetism
flattens in paramagnetic regions,
increases paramagnetism

**Magnetic phase separation drives pedestal formation**
Recap and applicability conditions

- Plasma tubes with an excess or defect of pressure are convected radially, depending on the plasma magnetisation.
- Phase separation occurs at the flux surface in which $j_q$ changes sign.

Under what conditions?

- *The seed pressure perturbation is strong enough to protrude above or below the background pressure profile.*
- *Plasma elements must be long enough to average out the $1/R$ variation of the vacuum field: $\lambda_{||} > qR$. Otherwise conventional, ballooning-like transport would drive short diamagnets towards low $R$.*
- *Collisionality/resistivity: the temperature must be high enough for particles to sample LFS and HFS before being scattered out of the tube.*
- *Edge pressure must be high enough to allow negative perturbations as well as positive.*
Caveats

- So far we have treated the plasma tubes as “test particles” with a magnetic moment.
- Ignored geometrical magnetisation from $j_{||}$.
- No evolution equation for tube magnetisation.
- The magnetic interaction between plasma elements and bulk plasma is quite complex, and our model very simple (too simple?)
- We hope that a more detailed calculation, up to second order on the spatial variation of $F = RB_{\text{tor}}$, can be carried out. Kind of neoclassical magnetisation, instead of classical.
- Much harder to do ...
Interchange instability\(^1\)

- present when a radial force acts equally on electrons and ions
- equivalent to the Rayleigh-Taylor instability in a fluid.
- magnetization gradient acting on magnetized plasma blobs replace "gravitational field" or "curvature".

\[
\gamma = \sqrt{\frac{g}{\lambda}}
\]

Magnetization interchange growth faster for high magnetisation, strong seed, low field & mass

\(\gamma = \sqrt{-\frac{1}{\bar{\rho}_{m,\text{blob}}} \frac{\tilde{\rho}}{B} \frac{d\bar{B}_\zeta}{dr} \frac{1}{\lambda_\rho}}\)

\(^1\)M.N. Rosenbluth and C.L. Longmire, Annals of Physics, Volume 1, Issue 2, May 1957,120
Suydam criterion for interchange instability


\[ \beta' \left( \frac{Rq}{r_s} \right)^2 \left[ \frac{B^2 \kappa_r}{\mu_0} \right] > \frac{q'^2}{4q^2} \]

magnetic shear opposes interchange of tubes driven by cylindrical curvature and \( \nabla \beta \)

Generalization:
add magnetization force to cylindrical curvature

\[ \beta' \left( \frac{Rq}{r_s} \right)^2 \left[ \frac{B^2 \kappa_r}{\mu_0} + \tilde{M}_z \frac{dB_{0z}}{dr} \right] > \frac{q'^2}{4q^2} \]

In magnetically mixed states \( \tilde{M}_z \frac{dB_{0z}}{dr} < 0 \)
magnetisation force adds to curvature, instability,
until the magnetic shear \( q' \) or the variation of \( dB_z/dr \) changes.
Evolution towards magnetic phase transition

As heating is applied, low pressure paramagnetic plasmas have degraded confinement, driven by low $\nabla p$

When sufficient heating is applied, $\nabla p$ grows until zero magnetization is obtained somewhere inside the plasma: $j_\theta = 0$

\[
\nabla p = j_\zeta \times B_\theta + j_\theta \times B_\zeta = 0
\]

Estimate critical pressure gradient as

\[
\frac{dp}{dr} = j_\zeta B_\theta = E_{\text{loop}} \eta_{\text{Spitzer}} B_\theta
\]

Need database of typical $\nabla p$, loop voltage, resistivity and $B_\theta$ to test predictions

or measurements of $j_\theta$

Explaining $T_e$ threshold for L-H transition via $\eta_{\text{Spitzer}}$ and associated pressure gradient ($\beta_\theta$) threshold
Experimental evidence? AUG

(10×) $j_\theta$ zero crossing at L→H transition


<table>
<thead>
<tr>
<th>Summary statistics (ms) for 10 discharges</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIME DELAY</td>
</tr>
<tr>
<td>$t_{j_\theta=0}$ → $t_{L-H}$</td>
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In a slow L-H transition followed by “type III” ELMs, $j_\theta$ remains diamagnetic after transition.

But at least one counter example has been found in a slow transition (still unpublished). More analysis needed, as well as more refined model.
Summary and comments

- First-principles model of plasma magnetization and magnetic phase transition as the basis for triggering confinement transitions.
- The magnetic state of the plasma determines convective motion of high and low pressure tubes.
- Paramagnetic plasma regions attract cold tubes, become more paramagnetic.
- Diamagnetic plasma regions attract hot tubes, becoming more diamagnetic.
- A pedestal structure builds up in the magnetic phase boundary.
- Magnetic boundary defines critical magnetization: $j_\theta = 0 \Leftrightarrow \nabla p = j_\zeta \times B_\theta$
- Magnetization force drives the magnetic interchange mechanism in closed field line region, similar to curvature interchange in SOL.